COMPUTER GENERATED HOLOGRAMS
Optical Sciences 627
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PART II: CHAPTER SEVEN
SIMULATION OF NON-FOURIER HOLOGRAMS

Introduction

In this chapter we consider the process of recording and reconstructing 3-D object information from a CGH. We will make the following simplifications for practicality.

- The first simplification is that wave propagation will be done in the parabolic approximation.
- The object structure will be modeled as lumped.
- We will begin by considering object internal scattering to obey the Born approximation. Later in the chapter we will relax this condition.

We will consider perspective changes in viewing the 3-D image reconstructed from the CGH first by discussing stereo vision and then by investigating the effects of linear phase factors introduced in the simulated reconstruction of the CGH's. We begin by limiting our calculations to a simple two-plane system.

The Two-Plane Self-Luminous Object

The object we are considering is self luminous, propagation is in the Born approximation, and has the form

$$u(x,y,z) = u_1(x,y,z+z_0) + u_2(x,y,z-z_0)$$

Propagation to the hologram is modeled as convolution with the pinhole wave,

$$u(x,y,z_{CGH}) = u_1(x,y,z+z_0) ** u_{ph}(x,y,z_{CGH}+z_0) + u_2(x,y,z-z_0) ** u_{ph}(x,y,z_{CGH}-z_0)$$

Let us compress the notation by suppressing the ***x-y*** dependence,

$$u(z_{CGH}) = u_1 ** u_{ph}(z_{CGH}+z_0) + u_2 ** u_{ph}(z_{CGH}-z_0)$$

Reconstruction the image is equivalent to operating on the CGH-plane complex amplitude with the convolutional inverse of the appropriate pinhole wave. Fortunately, we can save ourselves a good deal of effort by Fourier transforming the equation and converting the convolutions to multiplications

$$U(z_{CGH}) = U_1 U_{ph}(z_{CGH}+z_0) + U_2 U_{ph}(z_{CGH}-z_0)$$

The reconstructions at the two planes will be

$$v_1 = IFT\left[U_1 + U_2 \frac{U_{ph}\left(z_{CGH} - z_0\right)}{U_{ph}\left(z_{CGH} + z_0\right)}\right]$$

and

$$v_2 = IFT\left[U_1 \frac{U_{ph}\left(z_{CGH} + z_0\right)}{U_{ph}\left(z_{CGH} - z_0\right)} + U_2\right]$$

Adjusting the Viewing Perspective

When we view a three-dimensional object, we have many cues to its structure. One of the strongest cues is disparity, the difference between the views from our right and left eyes. This disparity is part of the group of cues that arise because the 3-D object looks different from different perspectives. The question that immediately arises is: How do we simulate this perspective change? To answer this question, let us look at the wave from a single, on-axis point, that has propagated to the CGH. The complex amplitude, in the parabolic approximation, is

$$u_{ph}\left(x, y\right) = e^{\frac{i\pi}{\lambda z_0}\left(x^2 + y^2\right)}$$

Next let's look at the wave that would arrive from a shifted object.

$$u_{ph}\left(x - x_0, y\right) = e^{\frac{i\pi}{\lambda z_0}\left[\left(x - x_0\right)^2 + y^2\right]} = u_{ph}\left(x, y\right) e^{\frac{i\pi}{\lambda z_0}} e^{\frac{-2\pi i}{\lambda z_0} x x_0}$$

We can effect a shift by multiplication with a linear phase factor and a constant phase. The amount of lateral displacement is dependent on the depth. Let's consider two different depths and lateral displacements which give the same linear phase factor. They can be related by

$$\frac{x_0}{z_0} = \frac{x_1}{z_1} \quad \text{or} \quad x_1 = \frac{x_0}{z_0} z_1$$

laterally. This means that higher spatial frequencies will appear at the edge of the CGH, and these frequencies will in linear proportion to the displacement.. Care is necessary in the sampling requirements calculation.

Depth Effects in Modulation Images

What does this change do to the False Images

$$T\left(\xi, \eta\right) = \sum_m \sum_n U_{mn}\left(\xi, \eta\right) V_{mn}\left(\xi, \eta\right) e^{-2\pi i\left(mM\Delta x\xi + nN\Delta y\eta\right)}$$

$$U_{mn}\left(\xi, \eta\right) = A\left(\xi, \eta\right) \operatorname{sinc}\left[mA\left(\xi, \eta\right)\right] e^{im\phi\left(\xi, \eta\right)}$$

$$V_{mn}\left(\xi, \eta\right) = B\left(\xi, \eta\right) \operatorname{sinc}\left[mB\left(\xi, \eta\right)\right] e^{in\theta\left(\xi, \eta\right)}$$

we let

$$\phi \to \phi - \pi\lambda K_u\left(\xi^2 + \eta^2\right)$$

$$\theta \to \theta - \pi\lambda K_v\left(\xi^2 + \eta^2\right)$$

$$U_{mn}\left(\xi,\eta\right) \to U_{mn}\left(\xi,\eta\right)e^{-i\pi\lambda\left(mK_u\right)\left(\xi^2+\eta^2\right)}$$

$$V_{mn}\left(\xi,\eta\right) \to V_{mn}\left(\xi,\eta\right)e^{-i\pi\lambda\left(nK_v\right)\left(\xi^2+\eta^2\right)}$$